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TACKLING THE RETAILER DECISION MAZE: WHICH BRANDS TO DISCOUNT, HOW MUCH, WHEN AND WHY?

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We propose a model that seeks the optimal timing and depth of retail discounts with the optimal timing and quantity of the retailer's order over multiple brands and time periods. The model is based on an integration of consumer decisions in purchase incidence, brand choice and quantity with the dynamics of household and retail inventory. The major contribution of the model is that it shows how the optimum depth and timing of discount varies with key demand characteristics such as consumer stockpiling, loyalty, response to the marketing mix, and segmentation. In addition, the optima also vary with key supply characteristics such as retail margins, depth and frequency of manufacturer deals, retail inventory, and retagging costs. The most valuable contribution of the model is that it can provide an optimal discount strategy for multiple brands over multiple time periods.

The optimization model runs on a user-friendly personal computer program. An application based on UPC scanner data illustrates the model's uses. Sensitivity analyses of the optimization model under alternative scenarios reveal novel insights as to how optimal discounts vary as a function of the key demand and supply characteristics.

(Optimal Promotions; Retailing; Consumer Response; Discount Timing; Mathematical Programming)

1. Introduction

Retailers face a complex problem with regard to optimizing promotions in the current environment. This is due to the large number of categories, the multiplicity of similar brands in each category and the numerous deals by manufacturers for each brand. While research on promotions has greatly increased (see Blattberg and Neslin 1990), two central questions have not been answered directly: "When exactly should a retailer discount the price of a brand?" and "How deep should that discount be?" Two related questions are, "When and how much should a retailer order to efficiently meet consumer response to the discounts?"

These four questions are intimately related and need to be answered together. For example, a retailer can spend a trade deal on deep, infrequent discounts or shallow, frequent discounts. Similarly, the retailer could stock up during a trade deal to either sell more with a discount, or sell less at a regular price while retaining a higher margin. Difficult as these decisions are for one brand, they become even more complex in a multi-brand and multi-period context.

By *discount* we mean the retailer's temporary cut in a brand's list price to consumers, which may be accompanied by an in-store *display* or newspaper insert called a *feature.*

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By *trade deal* we mean the manufacturer's temporary cut in price to the retailer. By *planning horizon* we mean the promotional cycle during which manufacturers plan a schedule of promotions.

Many past studies have addressed the issue of optimal frequency of discounts. However, the optimal timing of discounts is a more specific issue than frequency of discounts. Typically, timing implies which week a discount should be offered during a planning horizon, while frequency merely suggests how often to discount *on the average* in a planning horizon, or at market equilibrium. Because of the dynamics of consumer stockpiling and manufacturer dealing, determining the timing of discounts is crucial to the joint solution. For example, if consumer stockpiling is high, it may be better for a retailer to offer discounts in non-successive weeks. Solving the timing issue is also a more difficult problem because it involves dynamic optimization over multiple periods of a planning horizon.

The retailer's promotion has two characteristics that distinguish it from that of the manufacturer with which readers may be more familiar. First, the retailer is not interested in brand but in *category* sales and profits. So getting consumers to switch brands by itself is not profitable, unless margin differentials justify it. Second, the retailer's margins are affected by the manufacturer, and typically fluctuate over the planning cycle based on manufacturer deals. So timing of discounts is directly related to manufacturer deals.

We develop a model with these issues in mind. The model has three parts. (1) A consumer response model captures consumer response to retail promotions for brands within a category; this model is a dynamic three-stage model of consumer purchase that links category incidence given store visits, brand choice given incidence and quantity given brand choice. (2) A retailer model describes the dynamics of retail inventory in response to retail orders and consumer purchases. (3) An optimization model maximizes the retailer's profits over a planning horizon.

In the present form our model does not yet account for cross-category and interstore effects. This simplification enables us to solve other complexities in modeling. The model may apply to many grocery products, such as crackers, yogurt and ketchup, which are not typically used to generate traffic or whose sales are not intrinsically dependent on sales of other categories. But applications of the model to other categories would be useful.

The next section relates our work to the relevant literature. We then describe the formulation of the proposed optimization model. Subsequent sections cover the consumer response model, estimation issues, implementation of the optimization, sensitivity analysis, and a discussion.

2. Literature

With reference to our work, Table 1 classifies the promotions literature on two dimensions: (1) the managerial focus of the study, whether the promotion decision is the retailer's or manufacturer's, and (2) the research goals, whether to explain the economics of promotions, model consumer response, evaluate promotions, or optimize promotions.

A large number of economic models in both marketing and economics have addressed the issue of price discounts (see Table 1, row 1). The primary purpose of these models has been to explain why such discounts take place, or what factors motivate the frequency and depth of these discounts. In contrast, our model captures consumer response to a specific retailer's discounts to jointly optimize their timing and depth.

Numerous empirical studies have also tried to model consumer response to manufacturers' promotions (see Blattberg and Neslin 1990). With the growing popularity of scanner data, more of these studies have used choice, timing or quantity models with

TABLE 1
Classification of Research on Price Discounts

Goal:	Focus: Manufacturer	Retailer
Economic Explanation	e.g.: Blattberg et al (1981) Lal 1990 Jeuland & Narasimhan (1985) Narasimhan 1988 Rao (1991) Raju, Srinivasan & Lal (1990)	e.g.: Sobel (1984) Varian (1980)
Response	e.g.: Guadagni & Little (1983) Gupta (1988) Lattin & Bucklin (1988) Tellis (1988a)	Blattberg & Wisniewski (1989) Kumar & Leone (1989)
Evaluation	e.g.: Blattberg & Levin (1987) Abraham & Lodish (1987) Pedrick & Zufryden (1991)	Mulhern & Leone (1991) Inman & McAlister (1992)
Optimization Static Dynamic Multiperiod	 Neslin et al (1992)	Armstrong et al (1992) Vilcassim & Chintagunta (1992) This Study, Gupta (1993)

disaggregate household data (e.g., Guadagni and Little 1983, Gupta 1988, Tellis 1988a). The empirical portion of our study is inspired by these latter models. In particular, we model three components of choice like Gupta (1988). We model the dependence of quantity on choice like Krishnamurthi and Raj (1989), using the two-step approach suggested in Tellis (1988a). We model the interdependence of timing and brand choice like Bucklin and Gupta (1992). In incorporating aspects of each of these four studies, our consumer response model goes a little further than each of them individually, while the optimization addresses an entirely new dimension.

Some empirical models have tried to evaluate promotions to aid the manufacturer's planning efforts. For example, Abraham and Lodish (1987) developed PROMOTER, an automated system to evaluate manufacturer's promotions by clearly separating out incremental sales due to promotions from base sales and other sales fluctuations. Blattberg and Levin (1987) developed an alternate approach to evaluate manufacturer promotions that accounts for forward buying and inventory costs of retailers. Our model differs from these evaluation models in that we attempt to explicitly find the *optimal timing* and *depth* of retailer's price discounts relative to various scenarios.

In general, relatively few published studies have focused on the retailer's problem. One study by Inman and McAlister (1993) seeks to evaluate whether signaling a price discount is better than actually providing a discount. Our optimization may be considered a generalization of theirs in the sense that we allow for any point in the entire discount range to be optimal. In addition, we also allow for the dynamic effects in consumer response. A second study by Mulhern and Leone (1991) seeks to evaluate the profitability of discounts when accounting for substitution and complementarity between product categories. Our approach is more limited in that we do not consider cross-category effects. However, it is richer in that it provides the specific depth and timing of discounts given multiple trade deals and details of consumer response for each brand.

Recently four as yet unpublished studies have focused on the optimization of promotions (Armstrong et al. 1992, Neslin et al. 1995, Vilcassim and Chintagunta 1991, Gupta 1993). We briefly distinguish our work from these studies.

Armstrong et al. (1992) develop a model to optimize a retailer's order quantity as a function of the length of retail promotions. Neslin et al. (1995), based on response parameters known from the literature, develop a dynamic optimization over a system of equations that describe a manufacturer's trade-off between advertising and promotion. Vilcassim and Chintagunta (1992) develop a static optimization model of the retailer's pricing strategy based on a response model of consumer store visits, category incidence and brand choice. In contrast to these three studies, ours is a dynamic model that optimizes both the timing and depth of price discounts along with a retailer's order quantity in response to consumer decisions in incidence, brand choice and quantity over time.

Gupta (1993) addresses a similar problem to the one we do by using a similar framework. However, his study incorporates only consumers' brand choice and purchase incidence, while we also address purchase quantity. In addition, we carry out extensive sensitivity analyses to determine general norms to suggest how the optima can vary by demand and supply characteristics for multiple brands over multiple periods.

In conclusion, as Table 1 indicates, the literature on promotions is growing rapidly and has taken different perspectives. Yet, no study has developed a dynamic optimization model for the timing and depth of retail discounts and order quantity.

3. Optimization Model

Formulation of General Model

We now formulate a general model whose goal is to maximize cumulative retailer *category* profits over periods $t = 1, 2, \ldots, T$ within a planning horizon that may contain trade deals for one or more brands[1], by optimizing the depth and timing of discounts and retail inventory. This model is stated as a non-linear, integer, mathematical programming model with the following profit objective function:

$$\underset{\{\mathrm{Disc}_{jt}, O_{jt}, \delta_{jt}, \xi_{jt}\}}{\mathrm{Max}} \left\{ \sum_{j,t} M \cdot S_{jt} \cdot (\mathrm{Price}_{jt} m_{jt} - \mathrm{Disc}_{jt}) \right.$$

$$\left. - \sum_{j,t} (\xi_{jt} \cdot F_{jt} + h_{jt} \cdot (I_{jt} + I_{jt-1})/2 + \delta_{jt} \mathrm{Tag}_{jt} \right\}. \quad (1)$$

Our objective is subject to the following constraints:

$$\mathrm{Disc}_{jt} \geq 0 \text{ and integer}, \qquad \forall_{j,t}, \quad (1a)$$

$$O_{jt} \geq 0, \qquad \forall_{j,t}, \quad (1b)$$

$$\xi_{jt} = 0/1 \text{ integer variables}, \qquad \forall_{j,t}, \quad (1c)$$

$$\delta_{jt} = 0/1 \text{ integer variables}, \qquad \forall_{j,t}. \quad (1d)$$

$$I_{jt-1} = (I_{jt} + S_{jt} \cdot M - O_{jt})(1 - \xi_{jt}), \qquad \forall_{j,t \neq 0 \text{ or } T}, = 0 \qquad \text{for} \quad t = 0 \text{ and } T. \quad (1e)$$

$$O_{jt} = (I_{jt} + S_{jt} \cdot M)\xi_{jt}, \qquad \forall_{j,t \neq T}, = 0, \qquad \text{for} \quad t = T. \quad (1f)$$

$$(\mathrm{Disc}_{jt} - \mathrm{Disc}_{jt-1})^2 (1 - \delta_{jt}) = 0, \qquad \forall_{j,t}, \quad (1g)$$

$$(\mathrm{Disc}_{jt} - \mathrm{Disc}_{jt-1})^2 - \delta_{jt} \eta \geq 0, \qquad \forall_{j,t}. \quad (1h)$$

[1] This implies that a retailer is assumed to have full knowledge of future trade promotions within the planning horizon. This knowledge may be based on history and previous experience with manufacturers as well as manufacturer announcements.

In equation (1), the profit function equals the profit margin before inventory costs less inventory costs for the product category. Inventory costs include the fixed costs of placing an order, the average cost of holding inventory, and the costs for changing the retail price such as retagging shelves.

The optimization model involves four basic control variables for each brand j ($j = 1, 2, \ldots, J$) within the product category, for each period t ($t = 1, 2, \ldots, T$) over the planning horizon:

Disc_{jt} = discount levels for brand j, during period t,

O_{jt} = retailer order quantity for brand j, made at beginning of period t;

ξ_{jt} = integer order time indicator (=1 if an order for brand j is placed during period t, 0 otherwise),

δ_{jt} = integer price-change indicator (=1 if a price change was made for brand j during t relative to period $t - 1$, 0 otherwise).

In addition, we define the following components of the model:

S_{jt} = average sales of brand j per consumer household during period t, computed as a function of causal variables (including, Disc_{jt}) and stated in terms of brand choice, purchase incidence, and quantity response (see next section),

m_{jt} = retailer profit margin, stated as a percentage of regular retail price for brand j during period t, *before* any retailer discount and excluding inventory costs,

F_{jt} = fixed cost of ordering brand j during period t,

h_{jt} = cost per unit of holding inventory of brand j during period t,

I_{jt} = retailer inventory for brand j at the end of period t.

Tag_{jt} = cost of retagging shelves if a price change of brand j occurs during period t,

Price_{jt} = regular price of brand j during period t,

M = total household market size, and

η = a fixed numerical constant between 0 and 1 (e.g., 0.5).

Discussion of Model Constraints

In the optimization model formulation above, there are four basic constraint types as discussed below:

1. *Control Variable Constraints*. Constraints (1a–b) ensure the non-negativity of discounts and inventory levels for each brand j during each period t. They also define the timing of discounts and inventory orders (i.e., 0 implies absence and >0 implies presence). Integer constraints (1c–d) are defined to operationalize other model constraints (i.e., 1e–h).

2. *Retailer Inventory Constraints*. To fully capture retail inventory costs and refine our characterization of retailer profits, our model also considers the pattern of retailer inventories.[2] For simplicity, and without loss of generality, we assume that no lag exists between the time of order and receipt of inventory. Then, retailer inventory is dynamically updated for each brand j over each period t on the basis of order timing, order quantity, and consumer purchases according to the following relationship:

$$I_{jt} = I_{jt-1} + O_{jt} - S_{jt} \cdot M. \tag{2}$$

[2] Our literature section has referenced the few related promotion models that consider retailer inventory control (e.g., Armstrong et al. 1992, and Gupta 1993). Also see Blattberg and Neslin (1990, pgs. 459–461) for a discussion of inventory control issues as they relate to promotion strategy. The literature on inventory control models focusses primarily on production planning (e.g., see Johnson and Montgomery 1979 for basic approaches in this area) and does not consider the relationship of inventory to promotional strategy as we do here. Consequently, we do not refer to the latter literature in detail here.

The retailer inventory relationship (2) is incorporated within our optimization model by restating it as the backward-recursive constraints (1e) in order to avoid circularity. Thus, equation (1e) updates inventory levels over consecutive periods as well as to set beginning and ending retail inventory conditions. In our formulation, we assume that consumer demand is known (by the consumer response model to be described in the next section) and will always be met by the retailer (i.e. no stock-out conditions are allowed). This means that inventory cannot go below zero (or some desired safety stock level), at any time for any brand in the category. For simplicity, we assume that inventory for each brand j will be zero initially (at $t = 0$) and will again be zero at the end of the planning horizon (at $t = T$) in (1e). If desired, alternative settings (e.g., such as a specified safety stock level rather than zero) can readily be made in our model.

Equation (1e) indicates that when an order for brand j is made at the beginning of period t, (and consequently when $\xi_{jt} = 1$), the previous period's ending inventory level, I_{jt-1}, will appropriately be set to 0. That is, to minimize inventory costs given a known demand, a retailer need only order when inventory depletes to zero (or reaches the safety stock level) at the end of the prior period. Alternatively, $\xi_{jt} = 0$ means that no order of brand j is made during t and equation (1e) becomes identical to equation (2).

3. *Retailer Ordering Constraints.* Since our model optimizes profits over a given multi-period planning horizon, the retailer order quantity in our model is related to sales in future periods and profitability over the planning horizon. This allows us to capture the dynamics of retail inventory, including the potential forward buying and stockpiling by retailers to take advantage of lower prices during a trade deal or increased purchases in response to retail discounts. Furthermore, to insure that demand will always be met, we specify the order quantity relationship relative to sales in each period as equation (1f). Thus, in equation (1f), when no order of brand j is placed (i.e., when $\xi_{jt} = 0$), the order quantity O_{jt} at the outset of period t equals 0. Alternatively, when there is an order in period t (i.e., when $\xi_{jt} = 1$) then, according to equation (1f), I_{jt-1} will be zero. Therefore to satisfy demand for period t, the order quantity O_{jt} will be set to $I_{jt} + S_{jt} \cdot M$ as suggested by equation (2). Again, given our assumption that demand is known, a retailer will order when inventory drops to zero since this policy will minimize both holding and order costs.

Hence, equation (1e) and (1f) provide a combination of powerful constraints that allow the optimal order quantity solutions to be determined directly *from the constraints alone*. This provides a significant advantage in the optimization because it eliminates the potentially numerous order quantity variables that would otherwise have to be treated explicitly as control variables within the optimization. Indeed, our computational experience has proven the solution of an alternative formulation that seeks order quantity solutions without such constraints to be extremely time consuming.

4. *Tagging Cost Constraints.* When a retailer changes a price, it may incur costs for retagging the shelves, updating computer records or managerial oversight. We consider such price-change costs that may arise from offering or changing a discount by means of the integer constraints (1 g) and (1 h). These equations are designed to detect discount changes over consecutive time periods by appropriately setting the integer indicator variables δ_{jt} or each brand j over consecutive periods.[3]

[3] If no discount change takes place over successive periods for a given brand j (i.e., the difference between successive discounts *equals* zero), then, (1 g) will be satisfied with either $\delta_{jt} = 0$ or 1. However, $\delta_{jt} = 1$ will violate constraint (1 h) while $\delta_{jt} = 0$ will not. Hence, as should be the case, δ_{jt} will be appropriately set to 0 indicating no tagging cost for period t. Alternatively, if a discount change has taken place (i.e., the successive discount difference is *greater than* 0), then $\delta_{jt} = 0$ will violate equation (1 g) while $\delta_{jt} = 1$ will not. Moreover, both $\delta_{jt} = 0$ or 1 will satisfy constraint (1 h). Consequently, here δ_{jt} will be appropriately set to 1 indicating that tagging costs should be considered for period t.

4. Formulation of Consumer Response Model

The previous discussion noted that average sales of brand j per consumer household during period $t(S_{jt})$ was an integral part of the optimization model. This section describes the development of this sales response function on the basis of its model components.

Retailers generally focus on category sales. These sales may be considered an aggregate of sales from consumers who visit the store, which in turn may be considered a composite of three consumer decisions: category incidence, brand choice and quantity (e.g., Gupta 1988). Traditionally, researchers modeled sales at the aggregate level (e.g., Tellis 1988b). Since the advent of scanner data, researchers have increasingly modeled sales at the disaggregate level (e.g., Guadagni and Little 1983, Tellis 1988a, Pedrick and Zufryden 1991). We adopt the disaggregate approach for two reasons. First, it provides greater insight into various consumer choices and thus allows a decomposition of sales into brand switching, stockpiling and consumption. Each of these dimensions has a different impact on the optimization of the retailer's resources. Second, models on disaggregate data are probably less prone to biases or distortions than those on aggregate data, especially when modeling dynamic effects such as post promotion dips (e.g., Neslin and Shoemaker 1989).

We estimate the average sales per consumer per store visit for a given brand j as:

$$E[S_{ijt}] = E[Q_{ijt}|B, C, V]P_{ijt}(B|C, V)P_{it}(C|V) \tag{3}$$

Here, the subscripts i, j and t stand for consumer, brand and weekly time period respectively. $P(X|Y)$ = the probability of event X given Y; where the events V, C, B and Q represent a consumer's store visit, category incidence, brand choice, and the quantity purchased respectively. Thus, equation (3) implies that quantity purchased is conditioned on brand choice, which in turn is conditioned on category incidence, which in turn is conditioned on store visit. Note, however, that equation (3) describes the conditionality of events and not their temporal sequence, so that these events need not have occurred in any strict time sequence. Thus, because $P(X|Y)P(Y) = P(Y|X)P(X)$, the final outcome does not change if the conditionality were reversed. From equation (3), we estimate the average sales per consumer for a particular brand j as:

$$S_{jt} = v_t E[S_{ijt}] \tag{4}$$

where v_t = average number of store visits per consumer during t. For notational convenience, we ignore subscripts for the store. We do not model store visits as a function of marketing variables because for our available category (crackers) buyers are unlikely to choose the timing or type of store based on promotion in the category. However, because promotions can cause store switching in some categories (e.g., Kumar and Leone 1988), an extended model of store visits may be necessary for those categories.

The conditionality modeled among the choice, incidence and quantity events goes beyond Gupta (1988), who treated the three events as independent. We account for the dependence of incidence on the attractiveness of brand promotions (e.g., Bucklin and Gupta 1992) and the potential censoring bias of modeling only positive values of quantity (e.g., Krishnamurthi and Raj 1989, Tellis 1988a). As a result, our overall model of sales becomes a two-stage Tobit model (quantity and choice) nested on a binary logit (incidence) given store visits. Thus, our consumer response model extends the work of the above four individual studies.

The next three sections model each of the three events that compose a consumer's brand sales. The following section describes the pattern of consumer inventory and its impact on the consumer purchase process. (The measures for the variables follow the data description).

Incidence Model

We can consider the two events, category incidence and brand choice, as a nested logit model. Maddala (1983) showed that for this model, choice could be modeled as a regular multinomial logit conditioned on incidence, while incidence itself as a binary logit with an "inclusive value" that reflects the attractiveness of the category (due to promotions and other marketing mix). By a clever derivation, he showed that this inclusive value is equivalent to the log of the denominator of the choice model. If the coefficient of the inclusive value is significant, it implies that promotions and other activities that influence brand choice also make the category more attractive. Further, Maddala showed that sequential estimation, which uses certain values from one equation in a subsequent equation, provides unbiased estimates that have the properties of a simultaneous estimation. We adopt such a sequential estimation procedure, instead of the simultaneous one, because of the ease of estimation and convergence. So we model incidence as a logit model of the probability that a consumer i will purchase in the product category during period t *given* a store visit, thus:

$$P_{it}(C|V) = \frac{1}{1 + \exp - (b_0 + b_1 \mathrm{CatPur}_i + b_2 \mathrm{Inv}_{it} + b_3 \mathrm{Incl}_{it})}, \tag{5}$$

where:

CatPur_i = mean long term probability of consumer purchasing in the category

Incl_{it} = Inclusive value (or category attractiveness) which is equal to the log of the denominator of the multinomial choice model (6) below.

Inv_{it} = the units of category inventory held by consumer i at the start of period t,

b = vector $\{b_0, b_1, b_2, b_3\}$ of model coefficients to be estimated.

Note that while consumers are allowed to have different values on the independent variables, these variables are assumed to have common effects (the b vector of coefficients) on incidence across consumers. The latter specification ensures parsimony in estimation and is the one commonly adopted.

Choice Model

Following Guadagni and Little (1983) and others, we model brand choice as a multinomial logit model dependent on a set of causal variables. These variables include consumer brand loyalty, prior brand purchase in the category and the retail mix at that time period. Typically the latter include the brand's price, discount, feature and display (Gupta 1988, Tellis 1988a). Based on axiomatic foundations relating to the utility of each alternative brand and a Gumbel distributional assumption on the random component of an individual's utility, it can be shown (e.g., see McFadden 1974) that the probability that a consumer i will chose brand j within a category comprised of J brands ($j = 1, 2, \ldots, J$) during a given period t *given* a category incidence event may be expressed as:

$$P_{ijt}(B|C) = \frac{\exp(\beta X_{ijt} + \gamma_j \mathrm{Disc}_{ijt})}{\sum_{k \in S} \exp(\beta X_{ikt} + \gamma_k \mathrm{Disc}_{ikt})}, \tag{6}$$

where Disc_{ijt} = Discount level available to consumer i for brand j ($j = 1, 2, \ldots, J$) during period t,

X_{ijt} = Vector of causal variables that includes Brand Loyalty, Lagged Choice, List Price, Feature and Display (to be described under the section Measures),

γ_j = Parameter for discount variable for each brand j,

β = Causal variable parameter vector = $\{\beta_0, \beta_1, \ldots, \beta_v\}$,

and k = Index for brands in the set, S, of brand choice alternatives.

Note that Brand Loyalty captures cross-sectional, while Lagged Choice captures temporal consumer heterogeneity. Because it offers greater mathematical tractability, we prefer this clear separation by means of two terms rather than the definition of a single dynamic loyalty index as in Guadagni and Little (1983). All the estimated parameters are assumed constant across consumers to ensure parsimony in estimation. Note that each brand j has separate coefficients for the discount variables because response to discounts varies substantially across our brands and may be shown to influence the optimization.

Quantity Model

The expected quantity of brand j purchased by a consumer i during a time period t *given* the occurrence of a brand choice event is expressed as the following exponential model:

$$E[Q_{ijt}|B] = \exp(a_0 + a_1\text{Price}_{ijt} + a_{2j}\text{Disc}_{ijt} + a_3\text{Inv}_{it} + a_4\phi_{ijt}/\Phi_{ijt}). \qquad (7)$$

Here, the explanatory variables Price_{ijt}, Disc_{ijt} denote regular brand price, discount for brand j ($j = 1, 2, \ldots, J$) during period t, respectively. Inv_{it} is defined as the household product category inventory level at the end of period t. We do not include feature and display in this equation, because these variables draw attention to a brand's promotion in a multi-brand context but do not promote purchase of larger quantities. ϕ_{ijt} and Φ_{ijt} are the density and the distribution functions evaluated at $_{ijt}$, the perceived utility of brand j estimated by the choice characteristics X_{ijt}, subject to some error (μ_{ijt}).[4] Moreover, ϕ_{ijt}/Φ_{ijt} is the *hazard rate*, and accounts for the bias that may occur when consumers buy different quantities on different brand choices (e.g., Tellis 1988a). For example, consumers may buy large quantities of a cheaper brand of bar soap for regular use, but small quantities of a special premium brand for guests. The inventory variable is the only one in this model that accounts for consumer heterogeneity. As before, the parameters assumed to be constant across consumers.

Equation (7) is very tractable as it can be linearized by a log transformation and, assuming the errors are I.I.D. normal, can be estimated by regression. While an ordered logit may be a more appropriate formulation for equation (7), we find that a logarithmic transformation achieves the same goals with less complexity and easier interpretation.

Consumer Inventory Model

Our model incorporates the dynamic pattern of product inventory held by consumers during each period t, similar to prior studies (Lattin and Bucklin 1989, Gupta 1988, Tellis 1988a). We describe this pattern by updating the consumer's inventory at the end of the prior period (Inv_{it-1}) with expected category purchases and consumption (c_{it}) during the current period as follows:

$$\text{Inv}_{it} = \text{Inv}_{i,t-1} + \sum_j S_{jt} - c_{it} \qquad (8)$$

Because this dynamic inventory measure is an explanatory variable in our quantity (7) and incidence (5) models, it works as an important factor that captures the dynamic effects of purchase acceleration and stockpiling caused by promotions.

[4] The error is assumed to be independently, identically, and Gumbel distributed (e.g., see Tellis 1988a, pg. 137).

5. Estimation of Consumer Response Model

We now describe the empirical analysis in three subsections: data, measures and results.

Data

To estimate the consumer response model, we use IRI's scanner data for regular saltine crackers because the category typifies consumer purchases in a market that is characterized by heavy promotions. This database also offers the advantages of being more detailed and better documented than some other databases. We restrict our analysis to 16 oz. sizes of regular saltines, because sizes other than 16 oz. are few, of limited volume, and show almost no promotional activity.

The data cover the period from January 1984 to December 1985 and consists of an initial sample of about 1000 panelists in one city. To properly operationalize loyalty, we exclude light buyers, those who leave or join in mid-year and those who make no purchases in the store in four successive weeks. The latter are either erratic buyers of the category, or make purchases at outlets other than those in the study. The working sample consists of 501 panelists. When we factor in the number of purchases and the number of brands, we get 9845 observations.

The market consists of 3 national brands, Premium, Sunshine, Zesta, and several private labels. The city has 12 outlets belonging to either chains or independents. The modal number of private labels per outlet is 2. One private label in many stores is fairly popular and has a large share of market. Because our focus is on consumer response, which even for private labels is likely to be the same across stores, and because within each store only that store's private labels are available, we gave common names (SPL1 and SPL2) to the private labels in each store. We are thus left with 5 brands: 3 national and 2 private labels.

Measures

To facilitate reading, we do not use abbreviations for the variables, but capitalize the first letter of the name of a variable. We have two measures to capture the effect of consumers' preferences for current brand choice. First, we measure brand Loyalty as the share (measured from 0 to 1) of a consumer's purchases of a brand during an earlier holdout period of 72 weeks (we estimate the model on the latter 32 weeks). We use a longer holdout period than prior researchers to obtain more exact and stable measures of Loyalty, especially because crackers, with an average interpurchase time of 34 days, are not purchased frequently. Second, to capture any variety-seeking over brands and any changes in preference we also include a variable for Lagged-Choice (e.g., Lattin and Bucklin 1989).

Similar to brand loyalty, we define CatPur as the mean long run probability of category purchase in the prior 72 weeks of data. CatPur is related to but not identical to consumption. The reason is that aside from the rate of consumption, some households may buy more frequently because they shop more frequently, are less concerned about promotions, have less storage space or buy in smaller quantities to ensure product freshness. CatPur captures the basic cross-sectional heterogeneity in interpurchase times due to all of the above factors. Because CatPur is estimated on the prior 72 weeks, there is no circularity in using it to predict incidence in the 32 weeks of estimation data. In this sense, CatPur measures the "loyalty" or preference for certain frequency in buying crackers.

Figure 1 presents a typical price path of a brand in a store. The path tends to be bimodal within a 2–3 month span, with a high and then a low price occurring most often. We define the List Price (in dollars) as the higher modal price. The List Price increases over the year for most brands and stores, possibly because costs increase. We

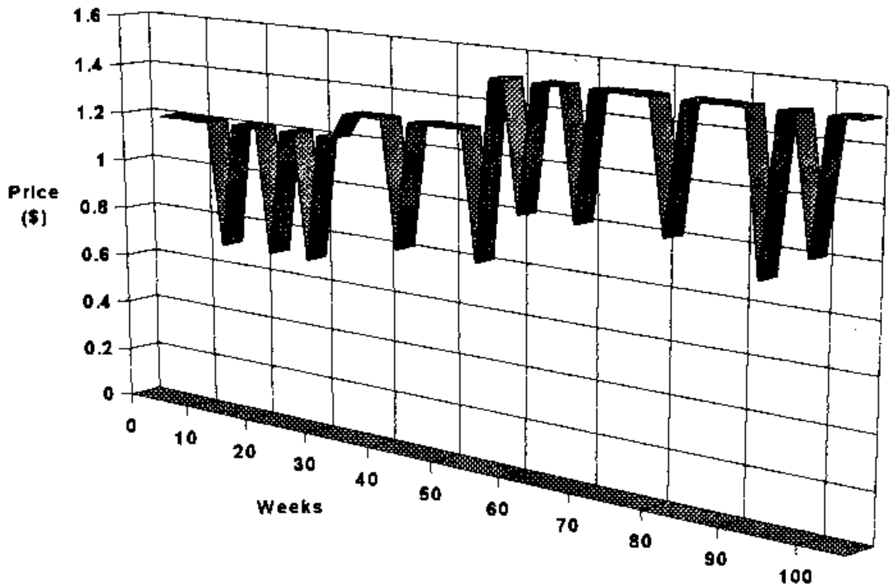

FIGURE 1. Time Path of Price for One Brand in One Store.

obtain the List Price by a computer program and visually inspecting the price for each brand and store. We found the latter necessary because of the price increases.

We define Discount (in dollars) as the List Price minus the actual price of the brand that day for each brand. We define Display and Feature as dummy variables which take on the value 1 if the brand was on display or featured on the day of the purchase, and 0 otherwise.

We define the consumer's inventory as the sum of the inventory of the last period, minus consumption during the last period, plus purchases, if any during the last period. We define the consumption rate as the sum of all purchases during the two-year period divided by the number of weeks and the market size (M). We initialized household inventory as the mean value obtained from our empirical data.

Again, while our measures may be imperfect, they have been used frequently (e.g., Guadagni and Little 1983, Lattin and Bucklin 1989, Gupta 1988, Tellis 1988a), and seem adequate for our purposes.

Results

Table 2 presents the estimates of the consumer response models. The two key variables that influence category incidence are the mean probability of a consumer buying in the product category (CatPur_i) and household Inventory (Inv_{it}). Because each household operates at a different level of inventory, a measure of inventory normalized over time for each household gave a better fit, as it better reflects temporal changes in inventory rather than heterogeneity across households. As expected, the effect of household inventory is negative. Both variables have high t-values, which indicates a small probability that the results are due to chance. The coefficient of the Inclusive value (Incl_{it}) is marginally significant indicating that the attractiveness of the category does influence incidence.

The model correctly predicts 44% of category purchases, 89% of non-purchases and 86% overall. Because cracker is a non-routine purchase, non-purchases exceed purchases, and the model predicts the latter better. Also, the data do not contain some important information that stimulates purchases of such infrequently purchased products. People probably buy for parties, picnics and other special occasions that are not recorded in the

TABLE 2
Empirical Results

Model:	Incidence		Brand Choice		Quantity	
Independent Variable:	β	t-stat	β	t-stat	β	t-stat
Category	12.50	13.2				
Purchase Prob.	−0.82	−11.3			−0.08	−4.6
Inventory						
Loyalty			3.19	10.9		
Lag-Choice			0.54	2.7		
Price			−1.14	−3.5	−0.47	−4.2
Feature			0.40	1.1		
Display			1.04	4.4		
Discount-Premium			4.01	3.2	.96	3.4
Discount-Zesta			1.22	0.6	−.76	−0.3
Discount-Sunshine			6.12	3.4	5.21	10.9
Inclusive Value	.01	1.7				
Hit Rates:	44%		72%			
True positive						
True negative	89%		92%			
True total	86%		88%			
Model Fit	$U^2 = 20\%$	$\chi^2 = 276$	$U^2 = 58\%$	$\chi^2 = 581$	$R^2 = 26\%$	$F = 32$

data. Given these considerations, the fit of the model is good though not excellent. Note that reversing our predictions (i.e., predicting a 1 for 0 and 0 for 1 would lead to a better hit rate of 56% for purchases, but a mere 11% for non-purchases and 14% for total purchases. With a U^2 of 20%, it compares well with results in the literature. For example, Gupta (1988) obtained a value of 6.7%; Bucklin and Gupta (1992) obtained values ranging from 18% for one segment to 20.6% for three segments.

Table 2 also presents the results of the brand choice model. In terms of t-values, brand Loyalty is the best predictor of brand choice; while Price and the promotion variables have similar but weaker effects. Note that the effect of Discount varies substantially by brand, being strongest for Sunshine. We dropped the Discount variables for the two private labels because they rarely offered discounts and the coefficients were never significantly different from 0. This model fits the data very well with a hit rate of 72% for brand choices, 92% for non choices and 88% overall.

Tellis (1989) argued that in an equilibrium where firms use discounts to discriminate between segments of switchers and loyals, lagged discounts should not have a negative effect on brand choice. As Neslin and Shoemaker (1989) have pointed out, such negative coefficients may be the result of aggregation biases. We tested and found no significant negative effect for lagged discounts in this category.

In the quantity model, Inventory and Price have strong negative effects and Discounts have positive effects, which vary strongly by brand. In terms of t-values, the effects of Price and Discount are a little stronger in the quantity models than in the brand choice model. (As expected, Display and Feature, even if included, are not significant in the model). These results are plausible, because Display and Feature are likely to influence which brand is chosen rather than how much of it. On the other hand, consumers are likely to choose both which brand to buy and how much of it to buy, based on its price and discount. The coefficient of the ϕ_{ij}/Φ_{ij} term is 0 indicating that quantity does not vary by brand chosen. This again is plausible for this category because we restrict our analysis to brands of 16 oz. saltine crackers that all serve an identical purpose and across which consumers are unlikely to vary quantities. The fit of the quantity model is fair with an R^2 of 26% and an F-value of 32.

6. Model Optimization

Based on our empirical estimates, we now discuss the implementation of the optimization, model and its solution on a PC.

Implementation of Optimization

In our formulation of the optimization model, discount levels (Disc_{jt}) are defined as integer variables. However, the resulting integer mathematical program is not as computationally tractable as a non-linear program based on continuous variables. Our experience has shown that rounding off continuous solutions from non-linear programs gives results that are very close to those from an integer solution. So, although we initially illustrate the application of the general integer program, for most sensitivity analyses we allow for continuous solutions of the discount variables. Aside from the increased tractability, the latter formulation more clearly reveals the insights from changes in the pattern of the optimal solution in response to changes in key parameters.

Despite the advantages from using constraints (1e–f), we found that our general formulation still could be computationally burdensome because the ξ_{jt} 0/1 variables may result in many feasible sequences of the retailer's inventory order. However, when we fixed the ξ_{jt} variables, by specifying 0/1 values to reflect a *given* sequence of orders, thus excluding them from the optimization, the model's solutions were quick. So, to make our optimization more tractable, we made simplifying assumptions about the order sequence. We assumed that retailers place simultaneous orders of each brand within the category on a consistent periodic basis (i.e., with a constant time between orders $\tau = 1$, 2, 3, or more weeks). Our procedure for obtaining a global solution then simply consists of solving the model for alternative τ values, by appropriately setting the ξ_{jt} variables, and then choosing the overall best of these solutions (with the best constant time between orders τ^*) based on the remaining control variables.

While this assumption makes for a much quicker optimization than one that includes a direct global optimization of the order sequence, we found that it produced *identical* solutions in a base case.[5] Thus, although the direct global optimization could have selected times between order intervals τ of variable lengths, it too determined that the optimal order time τ was *constant* (with $\tau^* = 2$ in this base case). Hence, our model shares the constant time between order interval property of a basic E.O.Q. inventory model. However, it generalizes the latter approach in allowing for variable order quantities at each order point.

PC Solution

Our optimization model can be solved quite efficiently on a PC. Our solution involved the use of spreadsheets in conjunction with the Solver optimization procedure in Excel for Windows. This procedure provides solutions to continuous non-linear programming models by using the steepest ascent method coupled with a Newton search. For the

[5] This is further discussed in the Sensitivity Analysis of Optimization Model section. Moreover, to insure that our model provides global, as opposed to local, optimal solutions, optimizations were conducted with various starting points. In all cases considered, we found the same unique discount solutions regardless of starting points.

integer programming models, Solver uses a "branch and bound" technique. Our PC approach is very easy to implement as the entire model can be set up in Excel by linking model equations on a spreadsheet, and solving in a manner transparent to the user. Another advantage of our approach is that it is easy to change model parameters or inputs on a spreadsheet and then resolve the model for purposes of sensitivity analyses.

Computations on a 486/33 MHz PC led to relatively quick solutions, especially with fixed τ values. Computational times were around 45 seconds for non-linear problems with continuous discount solutions, and about 2 minutes for problems with integer discount variables. Thus, a non-technical manager can effectively use the model on an interactive basis for practical applications.

The direct solution of the general integer programming model (with integer discounts, and 0/1 variables ξ_{jt} and γ_{jt}) took significantly longer to solve, with computation times around 45 minutes. Because our alternative approaches yielded similar results, the direct solution of the general model is not recommended for routine practical applications or extensive analyses of the model's sensitivity.

7. Sensitivity Analyses of Optimization Model

This section describes the sensitivity of optimal model solutions to different market scenarios defined by alternative supply and demand characteristics. These sensitivity analyses serve four goals. First, they explore the sensitivity of the model to changes in parameter values. Table 3 shows the ranges in parameter values used in the sensitivity analyses versus those found in the literature. Second, the analyses reveal certain relationships that when compared with actual practice, provide face validity for the model. Third, they reveal insights about how optimal discount and inventory policy change with changes in underlying variables. Fourth, to the extent that published parameters estimated in other categories lie within the range of our sensitivity analyses (see Table 3), the relationships identified here could serve as potential generalizations, or at least propositions for testing in other markets and categories or with other models.[6]

The following subsections first describe the optimization results for the base case. Then, we present sensitivity analysis from changing selected demand and supply characteristics.

Results for Base Case

In all our sensitivity analyses we consider a single retailer, stocking five brands, three national and two private labels, and planning promotions over a horizon of 12 weeks. Following the guidelines of the general manager of a large food chain in Los Angeles, we specified retail margins of 50% (of selling price) during trade deals and of 25% in the absence of trade deals for each brand in the category. Similarly, we also specified market size, order costs, inventory carrying costs, and prices consistent with the category in the Los Angeles area.

For this case, we consider the nonlinear integer programming model. Here, the optimization seeks integer discount solutions and considers retagging costs by means of integer (zero/one) constraints.[7] For this analysis, only Premium receives trade deals over periods

[6] However, to generalize our results, other market situations need to be considered since they may result in different effects.

[7] The cost of retagging, if suitably high, would discourage week to week changes in price unless they are very profitable. However, our informal survey of retailers indicated that such costs may not be high because prices are now being entered in the computer rather than marked on individual items. Thus, retailers may avoid making small changes given that retagging costs may now be lower and because they have no way of knowing the effect of such small changes.

TABLE 3

Ranges of Parameter Values Used in Sensitivity Analyses Contrasted to Values Reported in the Literature

Parameter	Values Used in Sensitivity Analyses	Values Reported in Literature	Corresponding Sources
Discount Coefficient (Choice model)	.5 to 4.5	2.34	Fader, Lattin & Little (1992)
		1.13* to 1.69*	Guadagni & Little (1983)
Inventory Coefficient (Incidence and Quantity models)	−2 to 0	−1.9 to −0.232	Bucklin & Gupta (1992)
		−0.516	Bucklin & Lattin (1991)
Loyalty Coefficient (Choice model)	1 to 6	3.30 to 3.50	Bucklin and Gupta (1992)
		3.40 to 3.50	Fader, Lattin and Little (1992)
		3.27	Bucklin & Lattin (1991)
		3.92	Tellis (1988a)
		2.78 to 3.92	Guadagni and Little (1983)
Inclusive Coefficient (Incidence model)	0 to 0.6	−0.020 to 0.410	Bucklin & Gupta (1992)

* For purposes of comparability, values were translated to \$/unit since the corresponding discount variable was defined in \$/oz in the Guadagni and Little (1983) study.

5 to 10 and only Premium may be discounted over the planning horizon. An iterative optimization, with each run using a different inter-order time, gave the same optimal order time of $\tau^* = 2$ weeks as a global optimization of the general model which allowed for variable τ values.[8] To further test the validity of a constant τ, we ran optimizations with different trade deals over a range of different costs of inventory. These generally led to a constant time between retail orders, supporting our assumption of this pattern. So, we set τ to 2 weeks for this and all subsequent sensitivity analyses.

Figure 2 shows some of the many possible outputs of the optimization that can aid a retailer. Note first that the optimal discount schedule matches the period of the trade deal, weeks 5 to 10. The optimal discounts are a little lower in the middle of the period probably due to lower response from consumers who have already stocked up on the brand. The optimal order quantity for retailers following the optimal two-week cycle, is correspondingly higher during the promotion period. So also are market share and profits. Note that the consumer's purchase incidence in the category peaks at the time of the first discount period 5, and then declines subsequently. This pattern is the result of consumers initially accelerating their purchase and stockpiling, followed by delayed purchases as they draw down their household inventories. Note that the purchase incidence begins to recover in the last 2 periods. Because we truncate the analysis at 12 weeks we are unable to show the full recovery.[9]

[8] It might be expected that forward buying should lead to a higher order quantity by the retailer prior to the promotional period and consequently to a longer time to the next order. However, our optimization results typically suggested a more periodic order pattern (see Figure 2). This is because increasing the time between orders, in this case, would lead to additional holding costs not offset by the reduction in ordering costs. However, in cases where we set relatively small inventory holding costs, the time between orders showed more variability. As might be expected, we found higher order levels suggested by the model at the outset and end of the trade period so as to meet demand increases from promotions and take advantage of the trade deal.

[9] Given that our present model is based on a finite horizon, the effects of promotions beyond this horizon are not considered. One way to consider such carry-over effects from promotions is to optimize the model over an extended horizon (e.g., to period 20) but only allow for promotions within the original planning horizon (e.g., 12 periods).

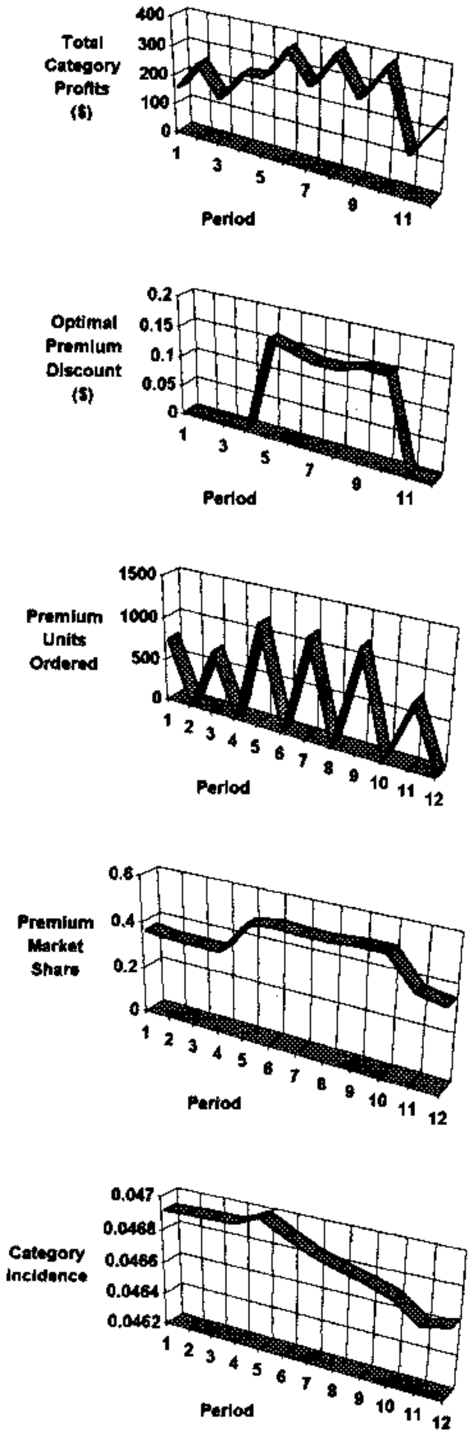

FIGURE 2. Illustrative Outputs for Base Case.

Results of Sensitivity Analyses

In this section, we first use the estimated parameters to forecast market response in our optimization model and develop the results for the base case. We then vary the model parameters to study the sensitivity of optimal discounts as we change certain underlying characteristics of the base case for both single and multiple brands' trade deals within a planning horizon.

1. *Effect of Multiple Trade Deals*. A useful aspect of our model is its ability to obtain the optimal timing and depth of discounts for multiple brands over multiple periods. We only consider the three national brands (Premium, Sunshine and Zesta) and omit the two private labels because the latter have discount coefficients not significantly different from 0. The following sensitivity analyses are all based on continuous rather than integer solutions of the discount variables and ignore retagging costs in order to isolate multiple brand effects. We now consider four cases based on the alternate scenarios for the trade deals: (a) Premium alone has a trade deal (Figure 3a); (b) premium, Sunshine and Zesta have partially overlapping deal periods (Figure 3b); (c) all three brands have fully overlapping deal periods (Figure 3c); and (d) all three brands have partially overlapping trade deals as in (b) but have different margins across brands (Figure 3d).

Case a—Figure 3a shows the optimum solution. Note first, that Premium should be on discount during its trade deal and at no other time (i.e., periods 5 to 10). The reason is that the response (coefficient) to Premium discounts together with its higher margin during the deal, make discounts profitable during, and only during the deal period.

First, two brands are not discounted simultaneously for the following reason. Whenever a brand is on discount, sales increase dramatically. Some of this increase may translate into higher revenue and profit to the retailer. However, the retailer also incurs opportunity costs. These costs arise from loyal buyers of the discounted brand availing of the discount even though they would have bought at the regular price. Now, when two brands are on discount, the group of brand switchers does not increase much, but rather splits up

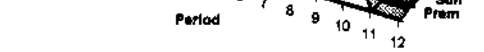

FIGURE 3a. Optimal Multibrand Timing and Depth of Discounts When Only Premium Is on Trade Deal.

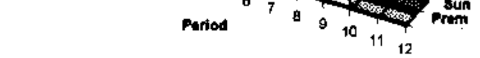

FIGURE 3b. Optimal Multibrand Timing and Depth of Discounts When Brands Have Partially Overlapping Trade Deal Periods and Same Retail Margins During Trade Deals.

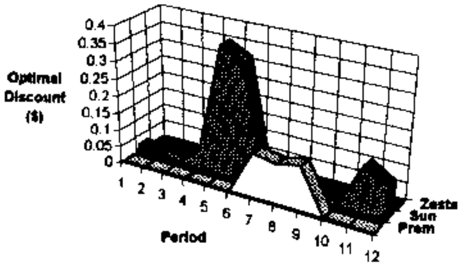

FIGURE 3c. Optimal Multibrand Timing and Depth of Discounts When Brands Have Completely Overlapping Trade Deal Periods and Same Retail Margins During Trade Deals.

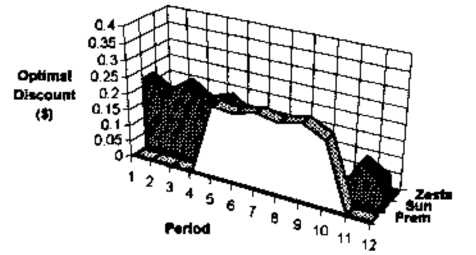

FIGURE 3d. Optimal Multibrand Timing and Depth of Discounts When Brands Have Partially Overlapping Trade Deal Periods and Different Retail Margins During Trade Deals.

among the discounted bands. So the retailer's profits are not much higher compared to when only one brand is on discount. However, the opportunity costs are higher because *now the loyals of each of the two brands buy on discount*. Thus, relative to individual brand discounts, simultaneous discounts of two brands is less profitable because it increases opportunity costs without increasing sales and revenues.

Second, because Sunshine has a relatively large response (coefficient) to discounts, it turns out to be an attractive brand to promote even in the absence of a trade deal. But because Premium is on discount in periods 5 to 10, and discounting is generally not profitable for two brands simultaneously, Sunshine is off discount during that period. This leads to an on/off discount schedule for Sunshine with discounts specified for this brand in periods 1 to 3 and 11 to 12. In contrast, note that Zesta should not be discounted at all because it has a relatively low response to discounts.

Case b—Figure 3b shows the results when all three brands have partially overlapping deal periods: Premium during periods 5 to 10 as before, Sunshine during periods 1 to 6, and Zesta over periods 7 to 12. Here, Figure 3b shows that the optimal solution follows an interesting discontinuous pattern of discounts.

First note, the relatively high discounts for Sunshine during *all* of its trade deal (periods 1 to 6), *even if* Premium or Zesta is on discount (periods 5 and 6), while moderate discounts for Premium only when Premium is and Sunshine is not on deal (periods 7 to 9). The reason is that Sunshine has so much more of a response to discounts than Premium and Zesta, that it is profitable to discount this brand *even* if the others are on deal. However, as in case 3a, the second wave of discounts for Sunshine (in periods 11 and 12) are smaller than those in the earlier periods (1 to 6). This may be explained by the absence a trade deal incentive for Sunshine after period 6. In contrast, despite the trade deal for Zesta, it does not receive any discounts because of its low response to discounts.

Second, in this case, note again that discounts for two brands never coincide even when trade deals overlap. The reason as explained above is that when sales increase come primarily from brand switching simultaneous discounts are unprofitable to a retailer.

Case c—Here, all three national brands are on trade deal during the same, completely overlapping, time periods 5 to 10. Figure 3c shows the corresponding optimal discount schedules in this case. Note that even though the trade deals *overlap completely*, the discounts are *completely staggered* for the reason offered above. Also, as before, Zesta is not offered on discount because of its low discount coefficient. In contrast, Sunshine is offered on discount, more steeply than Premium because of the former's higher response to discounts. As before, Premium is on discount only when it is on deal, while Sunshine is discounted even when it is not on deal.

Note however, that in apparent conflict to case b, when the deal periods completely coincide, Premium is on discount *even when* Sunshine is on deal. The reason is that Premium's discount coefficient renders discounts on Premium profitable only during a trade deal, while Sunshine's response make discounts profitable even when it is not on deal. Since simultaneous discounts tend to be unprofitable, the solution suggests trade deals for Premium only during its discount even *though Sunshine* is on deal, because Sunshine can be profitably on discount at other periods.

Case d—The growing importance of category management suggests looking at the effect of different retail margins across brands (Mulhern and Leone 1991). To illustrate this effect, we modify the scenario of Case 2. Instead of assuming a 50% margin for each brand, we now assume differential margins of 60%, 40% and 50% for Premium, Sunshine and Zesta respectively. Figure 3d shows the optimal pattern of discounts for the three brands. As before, despite the trade deal for Zesta, it does not receive any discounts because of its low response to discounts. Furthermore, Sunshine is discounted over periods

1 to 6 and during 11 and 12, as before. However, its optimal discount pattern is lower than that of the previous case due to its lower margin.

In contrast, due to its higher margin, Premium now has a markedly different optimal discount pattern. Not only are the optimal discounts higher for Premium than they were in Case 2 but they now extend over a longer period (periods 5 to 10 instead of 7 to 9 in Case 2). Note also that while Premium and Sunshine discounts never coincided before, the optimal solution in this case suggests that both Sunshine and Premium be simultaneously discounted (only) during periods 5 and 6. The reason is that the very high margin on Premium coupled with the very high response to discount of Sunshine now more than compensate for the opportunity loss of simultaneous discounts.

In summary, the above sensitivity analyses suggest that retailers should offer individual brand discounts: (1) during trade deals for brands with moderate response to discounts, (2) during off deal periods for brands with higher response to discounts, or (3) not at all for brands with low response to discounts. In contrast, retailers should not offer discounts on two or more brands simultaneously unless the margin or response to discounts on those brands are high enough to compensate them from the opportunity loss from many regular buyers availing of the discounts.[10]

The plausibility of these norms provides face validity for the model. However, the exact depth and timing of the discount and the specific brand to discount to maximize profits can only be ascertained by actually running the model for given market scenarios.

2. *Effect of Market Segments*. We next illustrate how market segments affect the pattern of optimal discounts. To obtain results useful for a retailer, panel data requires aggregation at some stage. Three broad approaches are possible.

First, one can aggregate the panel data to sales *prior to estimation*, then estimate a sales response model and carry out sensitivity analyses. The limitation with this approach is that coefficients may be biased (Neslin and Shoemaker 1989) and insights about consumers' choices (such as stockpiling, brand choice and quantity) are lost.

Second, one can first estimate the disaggregate model, but carry out sensitivity analyses by assuming that all consumers have the same mean values on the variables as in the data and the same response coefficients as those estimated from the disaggregate choice model. This approach avoids biases from data aggregation and retains insight about the stages of consumer choice. This approach is much faster because the sensitivity analyses are for an "average consumer" rather than for every consumer. This is the approach that we have adopted in the sensitivity analyses thus far. However, the second approach is not quite valid, if the distribution of consumer response is multi-modal (i.e., there are segments of consumers, each of which has distinct values on the variables and on the coefficients).

In that case we can use a third approach. We estimate the model *by appropriate market segments* (e.g., Kamakura and Russell 1989, Bucklin and Gupta 1992), and then carry out sensitivity analyses through a joint optimization of the weighted profits from the individual segments. Here, each segment is reflected by a consumer response function with different coefficients for loyalty and response to discounts. The relative size of the segments within the target market is reflected by the weights used for aggregation. We next explore the effect of such segmentation.

For illustrative purposes we assume two segments, loyals and switchers, of equal size. We differentiate these segments by contrasting their discount, loyalty and inventory

[10] Although most of our multibrand sensitivity analyses suggested this conclusion, we found a few cases (including Case d) where simultaneous discount periods were suggested when either or both Premium and Sunshine had relatively high retailer margins during deal periods.

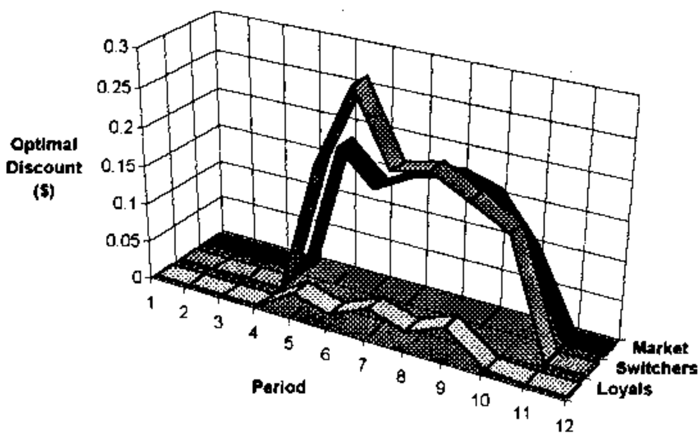

FIGURE 4. Optimal Premium Discounts in a Segmented Market.

coefficients.[11] Figure 4 shows the results of the sensitivity analyses. The curves labeled "loyals" and "switchers" are optimization runs assuming only loyals or only switchers. respectively. The curve labeled "market" is the result of the joint optimization described above.

Note first, that the solution suggests a much stronger pattern of discounts for switchers than for loyals. The reason is that it is less profitable to offer discounts to loyal consumers. The "market solution" seems strongly influenced by the optimal pattern for switchers, more so than an aggregate (not shown) computed on a weighted average of the solutions of each segment. Thus, if the market is segmented, the solution should be based on a *joint optimization* of the market formed by a weighted sum of the segments, rather than a posterior weighting of the discount solutions of the individual segments.

3. *Effect of Brand Dominance*. Many consumer markets are dominated by one or two brands with a large number of smaller national brands, private labels or generics. Should a smaller brand discount less or more relative to a larger brand? How should these discounts differ from the dominant brand's in terms of timing and depth? We explore these issues by varying the value of the loyalty variable. Recall that loyalty measures the share of a brand's purchases of the household over a prior hold out period. We find that dominant brands generally have a higher mean loyalty due to both stronger loyalty and more loyal consumers (Ehrenberg et al. 1990). For example, Premium has a mean loyalty of .47 while Sunshine has a mean loyalty of .07, which are the values used in the base case. Thus, to simulate the role of brand dominance, we contrast the optimal discounts from the base case with that obtained when Premium's loyalty is set to .07 and Sunshine's to .47.

Figure 5 presents the results. Here, the timing and pattern of discounts is similar for the two cases and is restricted to the trade deal period as in the base case. However, figure 5 shows very clearly that when brand dominance for Premium is higher the depth of discount is lower. This result is consistent with McAlister (1986). The reason for this result is that a brand with more loyal consumers has more to lose from these loyal consumers taking advantage of the discount than one with few loyal consumers. In other words, higher levels of brand loyalty make promotion less profitable for a given promotion parameter. Our results are also consistent with those of Raju et al. (1990) who suggest

[11] In our illustration, the loyals are assumed to have relatively low discount, high loyalty and low inventory coefficients. In contrast, the switchers are assumed to have relatively high discount, low loyalty and high inventory coefficients.

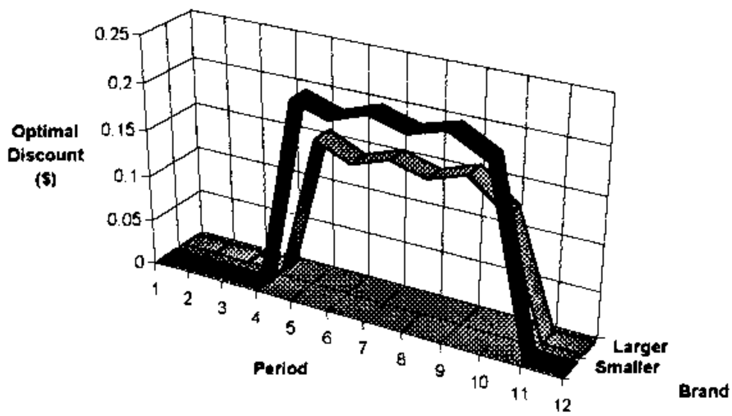

FIGURE 5. Optimal Premium Discounts Relative to Brand Dominance.

that a stronger brand should tend to offer a smaller discount, as compared to a weaker brand, depending on the specific values of brand loyalty.

4. *Effect of Retail Margins.* To focus on the impact of retail margins on the optimal discount, we varied retail margins for only Premium during the specified deal periods. Here, we sought to answer the following key question, "As trade deals provide a retailer with greater margins, should the retailer give greater discounts to consumers?"

Figure 6a provides the resulting optimal discount solution for Premium. As margins increase, the optimal solution implies that *greater* discounts should be given to consumers. Our sensitivity analyses also showed that the percentage of the trade discount passed through by the retailer to consumers during deal periods increases as retailer margin is increased. In particular, we found that the "pass through" percentage increases from 0% to about 35% as margins increase from 30 to 70%.[12] Interestingly, even though the retailer gives greater discounts to consumers, the retailer is also able to increase profits (Figure 6b) due to the higher margins per unit and the larger number of units sold at higher discounts.[13]

5. *Effect of Responsiveness to Discounts.* Other markets may be more or less responsive to discounts than the market we study. Alternatively, responsiveness to discounts may vary due to seasonal factors. For example, cracker consumption may be higher during winter months or festive occasions. To study the effect of responsiveness to discounts, we vary the discount coefficient for Premium in the choice model and raise the following question: "As brand choice responsiveness to discounts increases, should a retailer discount more?"

Figure 7 shows that as discount responsiveness increases (i.e., as the magnitude of the Premium discount coefficient increases), a retailer should indeed provide *greater* discounts to consumers, *but only during the deal period*. The reason is as follows. When the retailer discounts Premium, it incurs some opportunity costs as regular buyers of Premium take advantage of the discount. But it also makes a profit as brand switchers (regular buyers of other brands) buy Premium, if the margin on Premium during its discount is higher than that of the other brands not on discount. Now, as responsiveness to discounts increases, the latter profit increases relative to the opportunity costs. Thus, for markets that exhibit greater responsiveness to discounts, our results suggest that a retailer should

[12] The pass through percentage was computed from the ratio of (Retailer Discount × Consumer Sales on Discount) to (Trade Discount × Retailer Orders) over the promotion period.

[13] Note that the brand with the highest margin need not always be on discount because optimal discounts also depend on other factors such as the responsiveness to discounts and the extent of consumer stockpiling.

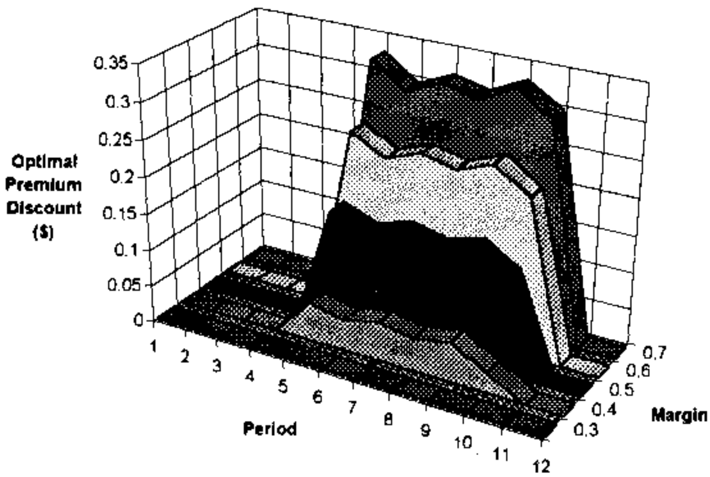

FIGURE 6a. Optimal Premium Discounts vs. Premium Retail Margin.

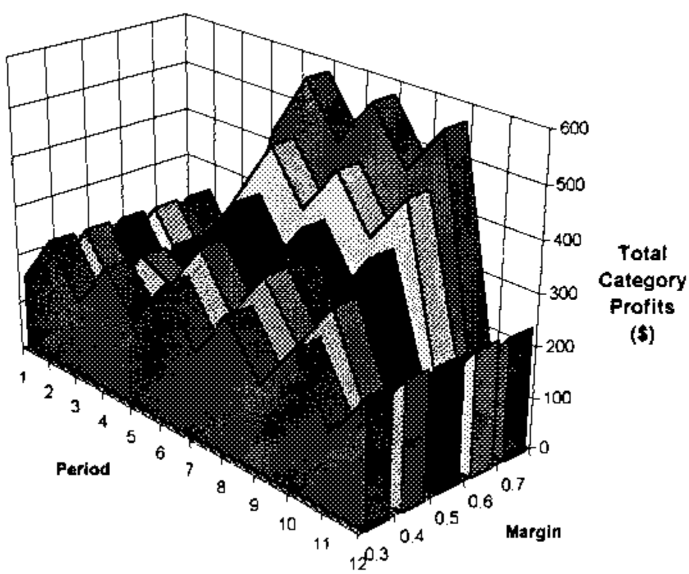

FIGURE 6b. Optimal Category Profits vs. Premium Retail Margin.

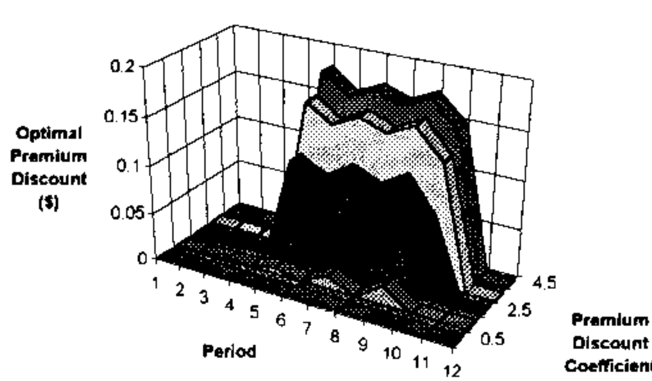

FIGURE 7. Optimal Premium Discounts vs. Responsiveness to Discount.

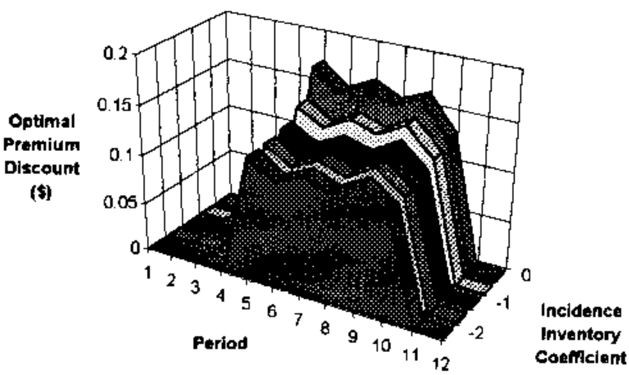

FIGURE 8. Optimal Premium Discounts vs. Responsiveness to Inventory.

provide higher discounts to consumers within the range of parameters found in the literature (see Table 3).

6. *Effect of Responsiveness to Inventory.* Other markets, and especially other categories, may be more or less responsive to the inventory held by consumers than saltine crackers in our market. For example, consumers may be more sensitive to inventory for products such as diapers and laundry detergent, both because of the inventory carrying costs and the out-of-stock costs. As a result, the likelihood of category incidence for such products is likely to increase (or decrease) substantially as home inventory decreases (increases). To study this issue, we vary the inventory coefficient in the incidence model and examine the impact on Premium. Correspondingly, we pose the following question: "As incidence responsiveness to inventory increases, should a retailer give greater discounts to consumers?"

Figure 8 shows the pattern of optimal discounts for Premium under alternative inventory coefficients. As incidence responsiveness increases (i.e., with larger negative inventory coefficients), a retailer should provide slightly *lower* discounts to consumers. The reason is that consumers are more likely to buy as household inventory decreases so that a smaller discount can achieve a larger response in terms of consumers' purchases of the brand.

7. *Effect of Responsiveness to Consumer Stockpiling.* When a retailer discounts a brand, consumers may tend stockpile it if they do not mind carrying the inventory. Such behavior raises an interesting question: "what is the effect of consumers' tendency to stockpile on the retailer's optimal ordering policy?"

To study this question, we ran the optimization while varying the inventory coefficient in the quantity model to reflect this tendency to stockpile.[14] The reason is that as consumers are more willing to stockpile, the quantity they buy will be less sensitive to their current inventory. Thus, the inventory coefficient in the quantity model should become less negative.

Figure 9 presents the results of this sensitivity analysis. Here, the optimal retailer inventory ordering policy for Premium varies as consumers' inventory parameter goes from -2 to 0, (i.e., as consumers are increasingly insensitive to their inventory and willing to stockpile.) Note that as consumers' willingness to stockpile increases, the retailer should order in higher quantities over time. Further analysis reveals that this result is

[14] These variations in the inventory coefficient of the quantity model only partially address the issue of consumer stockpiling. In our analyses, we found consumer stockpiling (as reflected by total household inventory carried) to be sensitive to variations in the inventory coefficients of *both* the incidence and the quantity models. Likewise, variations in the inclusive coefficient, which reflects changes in category attractiveness, were also shown to affect consumer stockpiling.

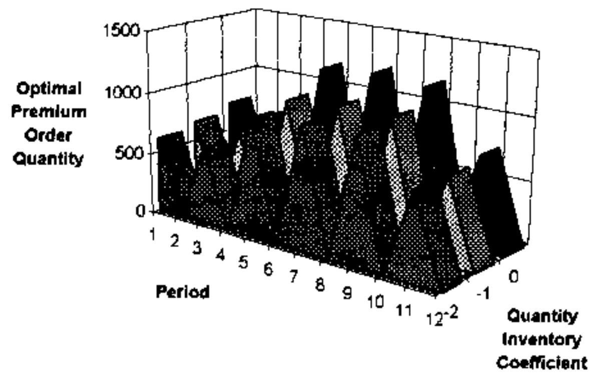

FIGURE 9. Optimal Premium Discounts vs. Responsiveness to Stockpiling.

because the retailer now sells more as consumers buy and hold more inventory. In effect, the consumers insensitivity to their inventory enables a retailer to order and sell more to consumers who also stockpile more. There is thus a transfer of inventory carrying costs from the distribution system to the consumer.

8. *Effect of Responsiveness to Loyalty.* In certain markets, choice may be guided more by brand loyalty than it is for our market. To study this issue, we vary the Premium loyalty coefficient in the choice model and raise the following question: "As consumer choice is increasingly influenced by brand loyalty should the retailer give greater discounts to consumers?"

Figure 10 shows the pattern of optimal discounts for different loyalty coefficients. Note that as dependence on loyalty increases (i.e., with larger loyalty coefficients), a retailer should provide substantially *lower* discounts to consumers. The reason is that as the loyalty coefficient increases consumers' choices are increasingly influenced by brand loyalty; thus, the retailer can reduce opportunity losses by offering less discounts to such consumers.

Note that although this result is similar to that obtained in brand dominance, both the factor and reason for the effect are a little different. In this case we vary the *coefficient for loyalty* which is common across brands, while in brand dominance we vary the *loyalty variable* itself which is different across brand. In this case, the reason for lower discounts

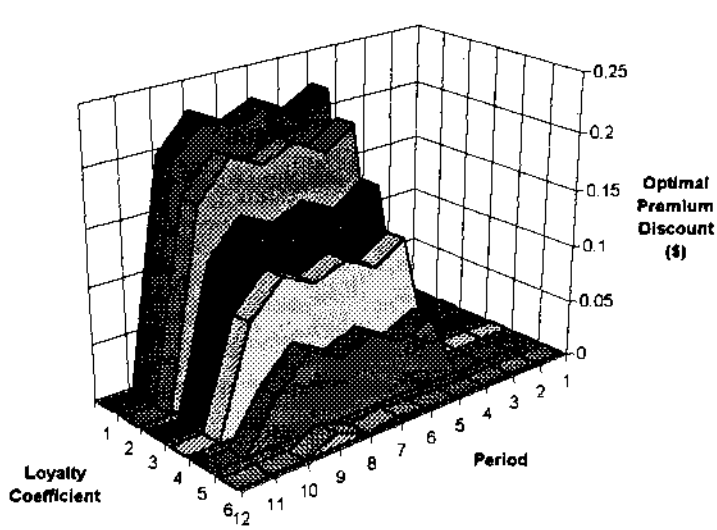

FIGURE 10. Optimal Premium Discounts vs. Responsiveness to Loyalty.

is to reduce opportunity loss by all consumers whose choices are more influenced by loyalty; in brand dominance the reason for lower discounts is to reduce opportunity loss by a larger segment of loyal consumers. This case reflects a retailer's differing discounts strategies between a suburban market that is influenced less by brand loyalty and an inner city market that is driven more by it. The brand dominance case reflects the differing discount strategies a retailer should adopt for a niche player (Sunshine) versus a dominant player (Premium).

9. *Effect of Responsiveness to Category Attractiveness.* How does the pattern of optimal Premium discounts vary as purchase incidence depends more on attractiveness in the category? Alternatively, how does the optimal pattern of Premium discounts vary as consumers time their purchases on promotions in the category? The inclusive value, which captures the dependence between incidence and choice, is the log of the denominator of the logit model. As such, it reflects the attractiveness of the category. Because from week to week, the change in promotions is the main influence on brand choice, promotions are an important component of the inclusive value. Thus by varying the inclusive value, we can capture the effect of this dependence of incidence on category attractiveness which is affected by promotions.

Figure 11 shows the results of this sensitivity analysis. As the inclusive value increases, the pattern of optimal discounts for Premium increases moderately. The reason is that as the inclusive value increases, consumer purchase of the category is driven more by the promotions of any of the brands. Thus, discounting in the category becomes much more effective and attractive.

8. Exploratory Validation

The broad results from our sensitivity analyses have face validity. While a formal validation would be useful (e.g., Blattberg and Neslin 1990), such an effort requires considerable information especially about retailers' costs, retailers' margins and manufacturers' deals in the market we studied. This information is unavailable to us if available at all. Further, even if we did obtain these figures for some current market, we would need to re-estimate the empirical model for the same market. Thus a formal validation is outside the scope of this paper. However, we do compare some key results with causal observation, our own data, and findings in the literature. Thus, our sensitivity analyses suggest the following key results among others:

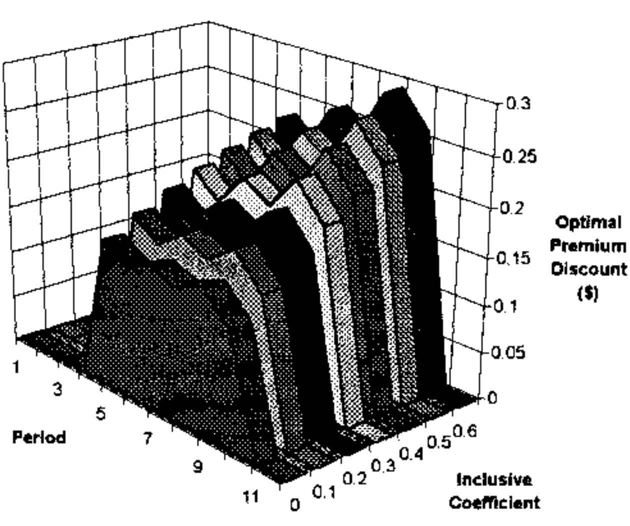

FIGURE 11. Optimal Premium Discounts vs. Responsiveness to Category Attractiveness.

H1. *Retailers should preferably not promote private labels. In none of our sensitivity analyses were promotions for private labels recommended because their response to promotions is so low.*

H2. *Retailers should not promote more than one brand at the same time, even if trade deals for these brands overlap. This result is visible in Figure 3 and the related discussion.*

H3. *Retailers promotions should increase as response elasticity increases. Figure 7 and the accompanying discussion supports this hypothesis.*

H4. *Retail margins are higher during trade deals. Figure 2 and the accompanying discussion shows that a retailer's total profits and thus profits per unit are higher during a trade deal because the retailer does not pass through all of the trade deal.*

H5. *Retailers should promote high share brands less than low share brands. We assume that brands have larger market shares because of a wider base of loyal consumers. This hypothesis then emerges from Figure 5 and the accompanying discussion.*

Note that H1 is consistent with Blattberg and Wisniewski's (1989) finding that the price cuts of private labels attract a narrower market than those of national brands. In addition, casual observation as well as data indicate that private labels are promoted much less than national brands, supporting H1. With regard to H2, casual observation as well analyses of our data show that retailers rarely promote two or more brands simultaneously. As for H3, Curhan and Kopp (1988) report that promotion elasticity was a major positive determinant of retail promotions in their study. With regard to H4: Chevalier and Curhan (1976) found that retail margins were consistently higher during trade deals. This was the result of their well known finding that retailers do pass through only about 33% of the trade deal.

Empirical evidence on H5, is not clear. Chevalier and Curhan (1976) indicate that high share brands may offer slightly higher trade deals than low share brands, but high ranked brands may offer slightly lower trade deals than low ranked brands. The authors did not provide evidence about retailers pass through by brand share or rank. Casual observation suggests that retailers may promote well known brands more, as a loss leadership strategy to attract store traffic. But other studies (e.g., McAlister 1986) do support our theoretical finding.

In conclusion, while a formal validation would provide greater insight regarding the validity of our model specification and results, our exploratory validation suggests that our results are generally consistent with our own data, casual observation and scattered empirical findings in the literature.[15]

9. Conclusion, Limitations and Future Extensions

We have proposed a dynamic planning model that gives the optimal timing and depth of retail discounts along with the optimal inventory policy. The model also provides other performance measures such as market share, inventory, and profits for each brand within a category over a given planning horizon. A unique aspect of our framework is that it is based on estimated response components that reflect consumer brand choice, quantity and category purchase behavior as well as retail inventory dynamics.

The major contribution of the model is that it shows how the optimum depth and timing of discounts vary with characteristics of the demand such as consumer stockpiling, loyalty, response to the marketing mix, and segmentation. The model also shows how the optima vary with key supply characteristics such as retail margins, depth and frequency

[15] We used follow-up data for the cracker category for one supermarket chain to compare the actual *average* depth of discount given by the chain's stores, over a period of 53 weeks, with the optimal discount solutions generated from our optimization model. Interestingly, this comparison showed relatively close coincidence for Premium and Sunshine. In contrast, while our multiple trade deal optimizations suggested no price reduction for Zesta, because of its relatively low discount coefficient, we found it significantly discounted by this chain. We would like to acknowledge and thank James Pedrick and Avu Sankaralingam of IRI for providing these data to us.

of manufacturer deals, retail inventory, and retagging costs. The most valuable aspect of the model is that it can provide an optimal discount strategy for multiple brands over multiple time periods. Such a strategy would not be apparent by mere inspection of the parameters.

The sensitivity analyses that we conducted suggested the following results: Retailers should give *higher* discounts to consumers as retail margins, consumer responsiveness to discounts and category attractiveness increase. Conversely, retailers should generally give *lower* discounts to consumers as consumer responsiveness to inventory and loyalty increase. Discounts should generally occur during a trade deal, with only one brand on discount. However, brands with strong response to discounts may be discounted at non-deal times and even when other brands are on discount. Conversely, brands with weak response may not be discounted even during a trade deal. In general, multiple brands should not be discounted simultaneously. In sum, our results provide insights about the pattern of optimal discounts as well as the sensitivity of optimal discounts to key model parameters. An exploratory validation supports many of these results.

From a practical standpoint, model solutions were easily obtained with a PC-based program. However, for model formulations with numerous integer variables, solutions may require considerable time. In addition, solution times may be significantly longer with increases in the number of brands in a category. Therefore, our model may be more difficult to apply in categories where retailers may stock numerous brands. Nevertheless, based on our work so far, our PC-based model appears to be a promising normative tool that should be useful for optimizing retail promotional strategy.

A major question is whether retailers would use models such as these which are based on disaggregate data. We think that this is very likely in the near future. To begin with, the popularity of disaggregate models should increase with diffusion through the literature and usage by syndicated data suppliers. For example, IRI now gives greater emphasis to the application of disaggregate models. Furthermore, better computers and software should also make the estimation and optimization of these models easier. Thus, in the future, such models should gain better acceptance.

From a practical standpoint, retailers could use such models on some key categories where promotion spending is high, and use that experience to complement the heuristics they now use. The results of the models should provide insights as to optimal behavior which may be subjected to experimentation or tests with other aggregate models. If disaggregate models are indeed less biased than aggregate models, the results from the former can be used as benchmarks for the latter.

With respect to future research, our general model can be extended to include other realistic aspects that may reflect actual retailer discounting practices as well as consumer response behavior. For example, when setting discounts, retailers may consider multiples of given price increments (e.g., 5 cents). Thus, rather than considering a continuum of integer values, discount levels might take on values of 5, 10, 15, 20 cents, etc. Moreover, a retailer may desire to consider a threshold discount level in setting his/her discount policy (e.g., see Inman and McAlister 1992). For instance, in order to create a more effective discount signal to consumers, a retailer may decide to either give no discounts or give discounts of at least 15 cents when discounts are set for particular brands. Such "either or" constraints may easily be considered by our model with 0/1 integer variables.[16]

In order to focus primarily on the timing and depth of discounts and simplify the model optimization, the feature and display variables were fixed at values based on our empirical data. However, a potential refinement for future research is the incorporation of other control variables to be solved jointly (e.g., regular shelf price as well as feature

[16] We have successfully implemented such model enhancements within our optimization model. However, in the interest of space, we refer the interested reader to Tellis and Zufryden (1992) for additional details and illustrations.

and display) within the optimization model. Clearly, such a refinement easily can be made within the context of the optimization model since these variables are already incorporated in our response models. However, a major difficulty is obtaining model solutions. This is because of the dramatic increase in the number of 0/1 control variables that are required to indicate the presence (or absence) of particular feature or display activities during a given period. Nevertheless, the latter problem is an important one that is the subject of continuing research by the authors.

Although our model considers retailer inventory decisions, our sensitivity analyses focused primarily on the decisions of timing and depth of retailer discounts. Our research clearly suggests that inventory decisions may play a critical role relative to the optimization of retailer profits. Thus, in future research, it would be of interest to explore more thoroughly the relationship of inventory costs and inventory ordering decisions in relation to optimal discount policies and retailer profit levels.

As previously noted, our model currently does not incorporate store visits as a function of promotions. With more comprehensive data, future research might involve an extension of our approach to explicitly model this aspect of promotion. Such a model could capture consumers' store switching, retailers' loss leader strategies and cross-category influences. Another extension could address the retailer's strategy of adopting an everyday low price versus the current strategy of high-low pricing.

In conclusion, our study addresses useful promotion management problems and suggests numerous areas for future research. Thus, we hope that our study will motivate further research towards the development of more comprehensive PC-based planning models for retailer decision-making.[17]

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